Evaluate the following assortment of definite and indefinite integrals.

(a) $\displaystyle\int_{ln(5)}^{ln(9)} e^{2r}dr$

Let $u = 2r$, thus $du = 2dr$; when $x = ln(9), u = 2ln(9)$ and when $x = ln(5), u = 2ln(5)$

Conclude that $\displaystyle\int_{ln(5)}^{ln(9)} e^{2r}dr = \frac{1}{2}\int_{2ln(5)}^{2ln(9)} e^{u}du = \frac{1}{2}e^{u}|_{2ln(5)}^{2ln(9)} = \frac{1}{2}(e^{2ln(9)} - e^{2ln(5)}0 = \frac{1}{2}(e^{81} - e^{25}) = \frac{1}{2}(81 - 25) = 28$

(b) $\displaystyle\int_{-4}^{4} |x|dx$

Break up the integral as: $\displaystyle\int_{-4}^{0} |x|dx + \int_{0}^{4} |x|dx = \int_{-4}^{0} (-x)dx + \int_{0}^{4} xdx = \frac{-x^2}{2}|_{-4}^{0} + \frac{x^2}{2}|_{0}^{4} = \frac{16}{2} + \frac{16}{2} = 16$

(c) $\displaystyle\int_{0}^{\pi/2} \frac{sin(x)}{1 + cos^2(x)}\,dx$

Let $u = cos(x)$, so $du = -sin(x)dx$. Then when $x = 0, u = 1$ and when $x = \frac{\pi}{2}, u = 1$

Conclude that $\displaystyle\int_{0}^{\pi/2} \frac{sin(x)}{1 + cos^2(x)}dx = -\int_{0}^{\pi/2} \frac{-sin(x)\,dx}{1 + cos^2(x)} = -\int_{1}^{0} \frac{du}{1 + u^2} = \int_{0}^{1} \frac{du}{1 + u^2} = tan^{-1}(u)|_{0}^{1} = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

(d) $\displaystyle\int \frac{cos(ln(y))}{y}\,dy$

Let $u = ln(y)$, so $du = \frac{1}{y}\,dy$. Then:

$$\int \frac{cos(ln(y))}{y}\,dy = \int cos(u)\,du = sin(u) + c = sin(ln(y)) + c$$

(e) $\int x^2 2^{x^3+1}\, dx$

Let $u = x^3 + 1$, so $du = 3x^2\, dx$

Then $\int x^2 2^{x^3+1}\, dx = \frac{1}{3}\int 2^{x^3+1} 3x^2\, dx = \frac{1}{3}\int 2^u\, du = \frac{1}{3}\frac{2^u}{ln(2)} + c = \frac{2^{x^3+1}}{3ln(2)} + c$

(f) $\int \frac{dx}{x^2 + 6x + 13}$

$$\int \frac{dx}{x^2 + 6x + 13} = \int \frac{dx}{x^2 + 6x + 9 + 4} = \int \frac{dx}{(x+3)^2 + 4}$$

Let $u = x + 3$, then $du = dx$ and $\int \frac{dx}{(x+3)^2 + 4} = \int \frac{du}{u^2 + 2^2} = \frac{1}{2}tan^{-1}(\frac{u}{2}) + c = \frac{1}{2}tan^{-1}(\frac{x+3}{2}) + c$

(g) $\int \frac{x^2 + ln(x)}{x} dx$

We can break up the integral: $\int \frac{x^2 + ln(x)}{x} dx = \int \frac{x^2}{x} dx + \int \frac{ln(x)}{x} dx = \int x\, dx + \int \frac{ln(x)}{x} dx = \frac{x^2}{2} + \frac{(ln(x))^2}{2} + c$

(h) $\int_0^4 \frac{x^3}{\sqrt{x^2+1}}\, dx$

Let $u = x^2 + 1$, then $du = 2x\, dx$. When $x = 0, u = 1$ and when $x = 4, u = 17$ Then:

$$\int_0^4 \frac{x^3}{\sqrt{x^2+1}}\, dx = \frac{1}{2}\int_1^{17} \frac{u-1}{\sqrt{u}}\, dx = \frac{1}{2}\int_1^{17} \frac{u}{\sqrt{u}}\, dx - \frac{1}{2}\int_1^{17} \frac{1}{\sqrt{u}}\, dx = \frac{1}{2}\int_1^{17} u^{1/2}\, dx - \frac{1}{2}\int_1^{17} u^{-1/2}\, dx = \frac{1}{2}(\frac{2}{3}u^{3/2} - 2u^{1/2})|_1^{17} = (\frac{u^{3/2}}{3} - \sqrt{u})|_1^{17} = \frac{17^{3/2}}{3} - \sqrt{17} - \frac{1}{3} + 1$$